3D MATERIAL MODEL FOR EPS RESPONSE SIMULATION

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ABSTRACT

In a country like the Netherlands, construction on weak and quite often wet soils is inevitable. The problems caused by these types of soils to road and railway structures demand an elaborate knowledge of the interaction between the foundation and the overlying pavement structure. The interaction aspects become even more important when issues of ultimate strength and long term behavior are concerned. For this reason, an extensive experimental, analytical and numerical investigation of the response of pavements founded on EPS has been undertaken at Delft University of Technology in Netherlands. One of the major goals of the investigation is the development and finite elements implementation of a generalized triaxial, strain rate sensitive, history and temperature dependent constitutive model. Explicit procedures have been formulated for the experimental determination of the model parameters.

Keywords: light-weight road structures, material modeling, structural behavior analysis

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INTRODUCTION

Soil conditions in considerable large areas of the western and northern parts of the Netherlands are characterized by extremely poor load bearing capacity and high groundwater level. Such unsuitable civil engineering characteristics on one hand, an extensive infrastructural network, on the other, force Dutch civil engineers to look for modern solutions. These innovative solutions have to provide an answer to ever growing traffic intensity and loading and lack of time for proper treatment of the soil (to improve its stability and limit the settlements). By minimizing the structure total deadweight and thus resulting settlements, the required construction time reduces significantly. Therefore the EPS geofoam approach is steadily finding its way into Dutch engineering practice (Duškov, Molenaar and Houben, 2001). Increased confidence and improved design procedures have enabled applications of EPS in Dutch motorways during the last years.

Without special precautions, the use of a low modulus material like EPS might induce problems with respect to the performance of the overlaying structure. The characteristics of EPS influence the overall pavement behavior. Due to multiaxial effects the yield stress of the material can be considerably lower than uniaxial tests show, Fig. 1.

Investigations to the long term durability of EPS in relation to varying environmental conditions, materials research on EPS, in-situ measurements and numerical analyses of the structural behavior of pavements with an EPS sub-base have been carried out (Duškov, 1997). Based on the research findings the current Dutch design guidelines for pavement design have been revised and optimized (Duškov and Houben, 2000). The research continues as ever. New challenges are, among others, improvement of roadbase bearing capacity by implementation of geogrids, development of lightweight railway structures and creating a better insight into structural dynamic behavior. Therefore adequate computer modeling is required.

This paper deals with the missing part in prior computer modeling (Duškov, Houben and Scarpas, 1998). Up to now EPS has always been represented by a linear-elastic model. This approach is sufficiently accurate in the most of cases, since occurring stresses in the EPS layer under a completed pavement structure are almost always within the linear-elastic region of the material. There are, however, critical situations when EPS geofoam is exposed to relatively high loading, in particular during the construction phase. If the EPS layer is overloaded to a certain extent then questions arise about its long-term performance under sub-base conditions, Fig. 2.
Furthermore, in dynamic analyses it is necessary to take into account the effects of plasticity. The EPS model described in this paper is the first step in an attempt to provide a realistic, unified, phenomenological approach for materials exhibiting strain rate dependent plastic deformations. The intention is to use experimental data from several sources to improve and verify the model.

CONSTITUTIVE MODELING

The theory of dynamic plasticity has been chosen as the most suitable constitutive framework for modeling the response of the material. By retaining the fundamental to classical plasticity notions of flow surface, decomposition of strains, hardening and/or softening, the theory has emerged as an attempt to provide a realistic, unified, phenomenological approach for materials exhibiting strain rate dependent plastic deformations. A modified version of the flow surface proposed by Gurson for porous isotropic materials is utilized for simulation of the triaxial stress response of EPS. This constitutive model has been implemented in the finite element system INSAP. Results of the utilization of INSAP for the investigation of the dynamic non-linear response of a road pavement will be reviewed in the last part of this contribution.

CONSTITUTIVE FRAMEWORK

The strain rate of the matrix material can be decomposed as:

\[ \dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p \]  

(1)

where \( \dot{\varepsilon} \) is a differential change in the total strain, \( \dot{\varepsilon}^e \) is a differential change in the elastic strain and \( \dot{\varepsilon}^p \) is a differential change in the plastic strain.

The associated flow rule of plasticity is defined as:

\[ \dot{\varepsilon}^p = \dot{\lambda} \frac{\partial f}{\partial \sigma} \]  

(2)

This equation can also be written as:
\[
\dot{\varepsilon}^p = \dot{\lambda} \left( \frac{\partial f}{\partial p} \frac{\partial p}{\partial \varepsilon} + \frac{\partial f}{\partial \sigma} \frac{\partial \sigma}{\partial \varepsilon} \right)
\]

\[
= \dot{\lambda} \left( -\frac{1}{3} \frac{\partial f}{\partial p} I + \frac{\partial f}{\partial q} n \right)
\]

with the standard Kuhn-Tucker conditions:

\[
\dot{\lambda} \geq 0, \quad f \leq 0, \quad \dot{\lambda} \cdot f = 0
\]  

(3)

The stress tensor can be written as:

\[
\sigma = -pI + \frac{2}{3} q n,
\]

(4)

where \( n = \frac{3}{2q} s \).

(5)

Evolution of plastic flow is determined by Prager’s consistency condition:

\[
\dot{j}(\sigma, \kappa) = 0
\]

(6)

in which \( \kappa \) is some measure of hardening/softening.

### HARDENING RESPONSE

Lab testing of EPS specimens is used to study the stress-strain behavior of the material, subjected to compressive loads. The compression tests were performed on dry cylinders with four different loading speeds, namely 0.2, 1, 10 and 100 mm/s (corresponding to strain rates of 4, 20, 200 and 2000 %/min) [Duškov, 1997]. The experiments have shown that, immediately after yielding, the strength of the material increases, Fig. 3.

![Fig. 3 Hardening response of EPS under compression](image)

The porosity of EPS plays an important role in the stress-strain behaviour of the material. When subjected to a compressive load the porosity will decrease, Fig. 4. When the stress is beyond the elastic limit the material is subjected to permanent deformations.
The evolution of the volume fraction of micro voids in the Gurson yield function is directly related to the permanent deformations. Inelastic response is controlled by the irreversible volumetric strain. The yield function and plastic potential in the material model are expressed as:

\[ f(\sigma) = \frac{q^2}{\sigma^2} \left[ 2q_1 \epsilon_p \cosh \left( \frac{3q_2 p}{2\sigma} \right) - q_3 \epsilon_p^2 \right] - 1, \]

(7)

where \( q = (3/2 : s : s)^{1/2} \) is the deviatoric stress,

(8)

and \( p = -\frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) \) is the hydrostatic stress (volumetric).

(9)

With aid of the definition of \( p \) we define the deviatoric stress:

\[ s = \sigma + pI. \]

(10)

\( I \) is the second order identity tensor. \( I = [1,1,1,0,0,0] \). The plastic stress in the material is represented by \( \sigma \). The parameters \( q_1, q_2 \) and \( q_3 \) are material dependent constants, to be experimentally determined from a standard triaxial test. Gurson’s yield function is capable to capture the effects of severe compaction because plastic strain growth under a hydrostatic state of stress can be modeled. This is due to the plastic loading function, which is closed on the hydrostatic axis, e.g. with a cap.
CYCLIC BEHAVIOR

Monotonic response is necessary to determine the bounds of ultimate material response, however in actual road engineering practice it is the cyclic response at working stress levels which is of importance and which after many load repetitions leads to material failure. EPS20 samples were subjected to representative uniaxial cyclic loads, Fig. 5. A short sinusoidal pulse was applied to simulate the load pulse shape caused by a moving wheel. A static load was used to simulate the dead weight of the overlaying layers.

![Figure 5](image)

**Fig. 5 Evolution of strains in dry EPS20 samples under combined static and cyclic stresses**

When subjected to cyclic stresses higher than 20 kPa the material shows permanent strains. This behavior is illustrated with the stress-strain curve in Fig. 6. Points A and B show the increase in deformation when the cyclic load is applied. Points C and D show the permanent deformation after a large number of load repetitions and when the cyclic load is removed.

![Figure 6](image)

**Fig. 6 Schematization of stress-strain curve**

The model is extended by isotropic hardening, which leads to uniform expansions, and by kinematic hardening, which models pure translations of the yield surface. When the material is subjected to cyclic loading, it has been experimentally observed that the stress-strain relation during unloading and reloading is not linear. The numerical simulation of this behavior via isotropic and/or kinematic hardening requires combinations of both hardening types. An improvement in the formulation is utilization of the so-called two-surface model. This involves the definition of another smaller surface, which can move under certain conditions within the bounds of the isotropic surface, Fig. 7. If the stress is inside the ‘small’ surface, the deformations are purely elastic. If the stress point is on the ‘large’ yield surface the deformations are plastic. Stress states outside this yield contour cannot exist. The characteristics of the nonlinear stress-path, the translation of the
small surface, are not necessary uniform for every load repetition, but can vary from $\omega_1$ at the first cycle to $\omega_\infty$ at the end of ratcheting.

![Graphical representation of two-surface model for plane-stress conditions](image)

**MATERIAL CHARACTERISTICS**

An extensive experimental investigation of EPS has been undertaken at Delft University of Technology [Duškov, 1997]. Most material parameters have also been studied at other institutes. Some model parameters, however, are not determined yet or can be improved. In order to calibrate the model a standard triaxial test, with a confinement phase and an increasing vertical stress, is needed. Simple compression tests on EPS show a clear strain rate dependant behavior. For large strain rates the E modulus reaches a constant value. Measurements by the Syracuse University show a relation between strain rate and E-modulus represented by:

$$E = 0.1094 \ln(\dot{\varepsilon}) + 4.1022.$$  \hspace{1cm} (11)

However, this equation implies that E keeps increasing while the strain rate increases. Therefore a new equation has been developed.

$$E(\dot{\varepsilon}) = 5 - 0.75e^{-0.01\dot{\varepsilon}}.$$  \hspace{1cm} (12)

The yield stress in the material can be determined via the vertical and horizontal strains. If the relation between them is not linear the material starts to respond inelastic. The yield stress can now be calculated with the maximum elastic strain. The relation between the vertical and horizontal strain will also be used to determine the Poisson’s ratio $\nu$. 


COMPUTATIONAL EXAMPLES

The constitutive model was implemented in the finite element code INSAP. Two numerical simulations of EPS under uniaxial compression are presented here. The model parameters, which were used, are not obtained from experiments. The model is able to take strain rate effects into account. Fig. 8 shows the stress-strain relation of the material under monotonic compression.

![Fig. 8 Stress-strain diagram of EPS under monotonic compression](image)

Fig. 8 shows the stress-strain relation of the material when subjected to uniaxial compressive loading and unloading. In this simulation stress-controlled cyclic loading was applied. Strain-controlled loading is also possible. This figure shows clearly that permanent deformations can develop, even for small stresses.

![Fig. 9 Stress-strain diagram of EPS under cyclic loading](image)
ACKNOWLEDGEMENTS

Stybenex is gratefully acknowledged for sponsoring the research into EPS geofoam.

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